

Note on reversibility of quantum jumps*

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Abstract

It has been recently proved that a quantum jump may be reversed by a unitary process provided the initial state is restricted by some conditions. The application of such processes for preventing decoherence, for example in quantum computers, was suggested. We shall show that in the situation when the quantum jump is reversible it supplies no information about the initial state additional to the information known beforehand. Therefore the reversibility of this type does not contradict the general statement of quantum measurement theory: a measurement cannot be reversed. As a consequence of this, the coherence of a state (say, in a quantum computer) cannot be restored after it is destroyed by dissipative processes having a character of measurement.

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The problem of preventing quantum decoherence in real systems became recently important in connection with the question about realizability of quantum computers [1, 2]. Decoherence as a physical phenomenon may be considered in a more general framework of theory of quantum noise [3]. However, decoherence as a physical process arising in the course of a quantum measurement, has interesting specific aspects. Some of them will be discussed here, with important conclusions about possibility to prevent decoherence.

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Mabuchi and Zoller has considered recently [4] a specific type of dissipation processes that may be characterized as a quantum jump i.e. disappearing of a photon, for example its absorption by a detector. The quantum jump may be described by an annihilation operator c (from the pair c, c^\dagger) as the transition $|\psi\rangle \rightarrow c|\psi\rangle$. It has been proved in [4] that the quantum jump can be reverted with the help of an unitary evolution provided the system has been before the jump in a state from a certain subspace. As a result, the initial state may be restored coherently, with the same phase relations as before the jump. This process was suggested as a possible mechanism for preventing decoherence, with possible application in theory of quantum computers.

A quantum jump is an example of a dissipative process. Therefore, the result of Mabuchi and Zoller [4] proves that some of dissipative processes may be reverted. Then the procedure providing the inversion of quantum jumps may serve as a method of preventing dissipation. We shall show however that the dissipation prevented in this way is not accompanied by obtaining new information and therefore cannot be identified with the decoherence arising in the process of a quantum measurement. It seems plausible that this is a general situation: decoherence cannot be inversed if any information is supplied by the process leading to this decoherence. This essentially restricts applicability of the procedure of Mabuchi and Zoller.

Our goal is therefore to show that the inversion of a quantum jump is possible only in the case when the jump supplies no information (additional to the information we had already before the jump), therefore it cannot be considered to be a measurement. The reversible dissipation is not a decoherence arising in the course of a quantum measurement.

1. Let the quantum jump be described by the annihilation operator c (from the pair of creation-annihilation operators c^\dagger, c). As it has been proved in [4], the quantum jump c may be reverted (i.e. the initial state of the system recovered) with the help of a unitary evolution, provided that the initial state belongs to some subspace of the state space \mathcal{H} .

This means that the action of the operator c on an arbitrary state from the specified subspace is identical with the action of some unitary operator U . Recovering of the initial state is then possible with the help of the evolution described by the operator $U^\dagger = U^{-1}$.

For the goal of the general theoretical analysis of this situation, we shall denote by \mathcal{H}_1 the subset of all vectors with this property, so that

$$c|_{\mathcal{H}_1} = U|_{\mathcal{H}_1}. \quad (1)$$

If the quantum jump c could give some information about the initial state, then it might be interpreted as a measurement. The contradiction with quantum measurement theory could arise in this case: the effect of the measurement on the system might be completely discharged by a certain unitary evolution. Our task is to show that this is not the case. We shall prove that, as a consequence of Eq. (1), the event of the quantum jump c gives no information about the initial state (other than the information following from the fact that the initial state belongs to \mathcal{H}_1). Therefore this event cannot be interpreted as a measurement.

For this end, we shall derive some properties of the states belonging to \mathcal{H}_1 and prove that the quantum jump gives no information about the initial state besides that this state had these properties.

First of all, it follows directly from Eq. (1) that for an arbitrary vector $|\psi\rangle \in \mathcal{H}_1$ the following equations are satisfied:

$$c|\psi\rangle = U|\psi\rangle, \quad \langle\psi|c^\dagger = \langle\psi|U^\dagger. \quad (2)$$

This means (because of unitarity of U) that the mean photon number for an arbitrary state from the specified subset, $|\psi\rangle \in \mathcal{H}_1$, is equal to unity:

$$\langle\psi|N|\psi\rangle = 1 \quad (3)$$

where $N = c^\dagger c$ is an operator of the photon number.

Let us expand the state $|\psi\rangle$ in a series of terms corresponding to definite photon numbers:

$$|\psi\rangle = c_0|\psi_0\rangle + c_1|\psi_1\rangle + c_2|\psi_2\rangle + \dots + c_n|\psi_n\rangle + \dots \quad (4)$$

where $|\psi_n\rangle$ is a (normalized) state with n photons. Then Eq. (3) reads as follows:

$$p_1 + 2p_2 + 3p_3 + \dots + np_n + \dots = 1 \quad (5)$$

with positive numbers $p_n = |c_n|^2$. For this equation being fulfilled, at least one of the numbers $p_1, p_2, \dots, p_n, \dots$ must be non-zero. Therefore, the state $|\psi\rangle \in \mathcal{H}_1$ cannot be vacuum. The expansion (4) must contain at least one non-vacuum component.

The quantum jump c diminishes the number of photons by unity. Therefore, the fact that the jump (the click of the detector) occurred, gives the information that the initial state has contained not less than one photon (could not be vacuum). However we know this already from Eq. (5). The event of the jump gives no new information and cannot be considered to be a measurement (provided we know already that the system has been in the subset \mathcal{H}_1 before the jump).

Of course, if two quantum jumps occur, this will supply some new information: that the number of photons was not less than 2. This could be a measurement. This however again leads to no contradiction, because the action of the operator c^2 (describing a double jump) is not equivalent to the action of a unitary operator even in the subset \mathcal{H}_1 . The double jump is a measurement, and its effect cannot be discharged by a unitary evolution. It is irreversible, in complete correspondence with general principles of quantum theory of measurements.

2. Let us suppose now that not only a single jump, but also a double jump may be unitarily reversed provided the system has been in the subset \mathcal{H}_2 before the jumps. This means that two unitary operators U_1, U_2 exist such that

$$c|_{\mathcal{H}_2} = U_1|_{\mathcal{H}_2}, \quad c^2|_{\mathcal{H}_2} = U_2|_{\mathcal{H}_2}. \quad (6)$$

Then the following relations may be readily derived for an arbitrary state from the subset, $|\psi\rangle \in \mathcal{H}_2$:

$$\langle\psi|N|\psi\rangle = 1, \quad \langle\psi|N(N-1)|\psi\rangle = 1. \quad (7)$$

Using the expansion (4), we may rewrite the same in the form

$$\begin{aligned} p_1 + 2p_2 + 3p_3 + \dots + np_n + \dots &= 1 \\ 2p_2 + 6p_3 + \dots + n(n-1)p_n + \dots &= 1. \end{aligned} \quad (8)$$

It is seen from Eqs. (8), that there is at least 2 photons in an arbitrary state of the subset \mathcal{H}_2 (i.e. such a state cannot be a superposition of the vacuum and the 1-photon state). Therefore, neither a single, nor a double jump give no additional information in the case when the effects of both a single jump and a double jump can be unitarily discharged. the single and double jumps are not in this case measurements.

It is evident that the same consideration is applicable also to the case of a multiple jump with an arbitrary multiplicity.

3. Quantum jump that means for example a click of a detector is considered usually as a sort of measurement in the sense that it supplies a new information. It has been shown above that the quantum jump may give no information if something is known about the initial state. In this case a measurement supplying nontrivial information might take place in the preceding step when the initial state had been prepared. This step should contain projection from the complete space of states \mathcal{H} onto some subspace belonging to \mathcal{H}_∞ (or \mathcal{H}_ϵ if the situation with double jumps is considered) . After this preliminary measurement the quantum jump (or double jump) gives no new information. One can formally say that such a jump is a measurement, but it should be clearly understood that this measurement gives no additional information.

This is not at all astonishing and is in fact common in quantum theory of measurement. Indeed, the text-book example of a quantum measurement is the measurement of an observable A with a discrete spectrum. If we have a series of repeated measurements of this type (beginning from an unspecified state), then only the first measurement supplies a non-zero information giving the measurement output a_i . All subsequent measurements of A will give with certainty the same result. The situation is quite analogous to the above discussion of quantum jumps if the stage of the preparation of an initial state is taken into account.

4. The above arguments support the general statement about irreversibility of quantum measurements (in the case when the initial state is not specified). This is important in the context of quantum computers and other devices depending on coherent character of their evolution.

One may hope to prevent decoherence resulting from dissipative processes, applying some or another correcting procedures, for example those proposed in [4]. It is shown in [4] that the initial state may be coherently restored after a certain dissipative processes. However the arguments of the present paper (apparently applicable to a more general situation) demonstrate that the restoration of the coherence is not always possible.

The class of dissipative processes leading to the irreversible decoherence may be specified by the concept of measurement or information. The restoration of the coherence turns out to be impossible if the dissipation is accompanied by obtaining information about the state

of a quantum system.¹ This essentially restricts the circle of situations in which recoherence is in principle feasible. This resulting restriction should be taken into account together with other principal difficulties in creating quantum computers [2].

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¹Of course, we do not necessarily mean a measurement arranged on purpose. Instead, it may be an interaction with the environment (reservoir) that results in recording the information in the state of the environment even if nobody is interested in this information.